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SU(4) Breaking and the New Particles: Some Applications*

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The ψ particles, discovered in e^+e^- production and annihilation, are now interpreted as compound states of a charmed quark and its charge conjugate.

The existence of a fourth quark, already conjectured many years ago by Bjorken and Glashow in analogy with leptons, is not unexpected since it has been advocated by Glashow, Iliopoulos and Maiani in connection with the current-current Lagrangian of weak interactions: there it has been shown the usefulness of introducing a new quark, with the same charge as the proton quark u. The introduction of this fourth quark leads to enlarge the internal symmetry group SU(3) of hadrons into SU(4) and therefore to complete the list of well-known hadrons with the presumably existent "charmed" mesons and baryons. Note that very recently SPEAR obtained good evidence for the particles D^0 and D^+, the pseudoscalar mesons supposedly constituted with $c\bar{u}$ and $c\bar{d}$ respectively (we will denote u, d, s, c the up, down, strange and charmed quark respectively). During the writing of this talk we just learned that physicists at Fermilab (Experiment E87) observed a peak at 2250 MeV/c^2 in the effective mass distribution of $\bar{\Lambda}\pi^-\pi^-\pi^+$ produced in the reaction $\gamma + \mathrm{Be} \to \bar{\Lambda} + \mathrm{pions} + \ldots$, and an indication of a state near 2500 MeV/c^2 decaying into $\pi^\pm + (\bar{\Lambda}\pi^-\pi^-\pi^+)$. These results can be interpreted following B. W. Lee, Quigg and Rosner as decays involving charmed 1/2 and 3/2 charmed antibaryons $\bar{C}$ and $\bar{C}^*$ (constituted with one c and 2 u, d quarks).

Moreover, the high value of the mass of the ψ particles compared to the 1^- vector mesons (or of the mass of $D^0(1855)$ or $D^+(1870)$ compared to the mass of the pseudoscalar mesons) is a clear hint that the SU(4) symmetry is strongly broken.

Here we will present, in the context of the breaking of SU(4) two different exercises, the corresponding preprints of which already appeared and will be published in Lettere al Nuovo Cim. The first one, entitled "The Weak Charges of the Charmed Particles" is a result of a collaboration with F. Buccella, A. Pugliese and A. Sciarrino, while the second one, entitled "Relativistic Bound States: A Mass Formula for Vector Mesons" has been done with J. L. Richard. The reader could find in these two papers all the necessary references that we will not list here, being limited in the number of pages.

A. THE WEAK CHARGES OF CHARMED PARTICLES

If the internal symmetry group SU(3) is enlarged into the group SU(4), then the classification group $SU(6)_W$ is enlarged into $SU(8)_W \supset SU(4) \times SU(2)_W$. More precisely, adding the orbital part $O(3)_L$ we could consider the classification group of hadrons $SU(8)_W \times O(3)_L$ or, as we proposed last year during the IVth Colloquium in Nigmegen the classification group $SU(8)_W \times SU(3)_{GOM}$ (GOM = generalized orbital momentum).

We propose to give an expression of the vector and axial vector charges of hadrons. The vector charges Q^i (i = 1, ..., 15) and the axial vector ones Q^i_5 are supposed connected, at $P_z \to \infty$, to the corresponding generators of the classification group $A\left(\frac{\mu_i}{2}\right)$ and $A\left(\frac{\mu_i \sigma_2}{2}\right)$ by a unitary transformation V.

Here we assume that the operator V acts on a system of quarks as the product of a set of commuting unitary operators acting on a single quark. As we had the opportunity to explain this during the IIIrd Colloquium in Marseilles in 1974, the Melosh transformation V can be seen as the Wigner rotation between the quark rest frame and the hadron rest frame. Following this approach the V operator can be easily written:

$$\begin{cases} Vq_i^{\uparrow}(\vec{k}, x) = \cos\hat{\theta}_i(\vec{k}, x)\, q_i^{\uparrow} + \dfrac{k_x + ik_y}{k_\perp} \sin\hat{\theta}_i(\vec{k}, x) q_i^{\downarrow} \\ Vq_i^{\downarrow}(\vec{k}, x) = \cos\hat{\theta}_i(\vec{k}, x)\, q_i^{\downarrow} - \dfrac{k_x - ik_y}{k_\perp} \sin\hat{\theta}_i(\vec{k}, x) q_i^{\uparrow} \end{cases}$$

It follows that the matrix elements of the charges concerning the lowest meson and baryon multiplets of SU(8) are proportional to the corresponding generators of the same algebra by an amount characteristic of each charge. This factor is cos $(\hat{\theta}_i \mp \hat{\theta}_j)$ for the nondiagonal vector and axial vector charges respectively; cos $(2\hat{\theta}_i)$ for the diagonal axial charges, while the diagonal vector charges (T_3, Y, C) are not renormalized, since V does not change the internal quantum numbers of the quarks.

As already checked by F. Buccella, F. Nicolo and C. A. Savoy, this ansatz has worked reasonably well in connecting the different effects of SU(3) breaking on semi-leptonic decays of the baryon octet; note in particular that a different normalization factor for the strangeness changing and strangeness conserving axial charges has practically the same effect as introducing two different angles for the Cabibbo rotation of the vector and axial vector weak currents.

Among the predictions which can be drawn from this study, let us note that for the strong decays of the charmed particles into a pion plus another charmed particle, one predicts the same coupling constants obtained with the strange quark in place of the charmed one:

$$\langle D^{*+} | Q_5^3 | D^+ \rangle = \langle \overline{K^{*0}} | Q_5^3 | \overline{K}^0 \rangle \quad .$$

In a similar way all the matrix elements of the charge Q_5^3 between the 20, $1/2^+$ and 20, $3/2^+$ states of the <u>120</u> of SU(8) (baryonic states) are all given in terms of one parameter.

An extension of PCAC to caons allows one to obtain from $\langle F^{*+} | Q_5^{6-i7} | D^+ \rangle = - \langle D^{*+} | Q_5^{6+i7} | F^+ \rangle$ the corresponding equality for the amplitudes for: $F^{*+} \to D^+ + K^0$ and $D^{*+} \to F^+ + \overline{K^0}$. More in general all the emissions of caons within states of the <u>63</u> and <u>120</u> are obtained in terms of only one parameter given by $K^* \to K + \pi$. To get a quantitative idea of the renormalization parameters involved it is appropriate to use the phenomenological information available about the V operator. In the formalism adopted, one is led to write:

$$\begin{cases} Vq_i^{\uparrow}|\text{ground state}\rangle = \cos\theta_i\, q_i^{\uparrow}|\text{ground state}\rangle + \sin\theta_i\, q_i^{\downarrow}|L_z = +1\rangle \\ Vq_i^{\downarrow}|\text{ground state}\rangle = -\sin\theta_i\, q_i^{\uparrow}|L_z = -1\rangle + \cos\theta_i\, q_i^{\downarrow}|\text{ground state}\rangle \end{cases}$$

where now the θ_i's are numbers and the state $|L_z = \pm 1\rangle$ is the same for each quark.

The mixing angle of the ordinary quarks can be obtained from the strangeness conserving and changing beta decays:

$$\theta_u = \theta_d \simeq 20^\circ \qquad \theta_\lambda \simeq 28^\circ$$

To determine θ_c, one may assume that also for charmed particles the matrix elements of the magnetic moment are proportional to the ones of the axial charges (which was a successful hypothesis for the baryon octet). Then we obtain the following general form for the magnetic moment operator:

$$\vec{M} = \mu_0\left(\frac{2}{3}\cos 2\theta_u \vec{S}_u - \frac{1}{3}\cos 2\theta_d \vec{S}_d - \frac{1}{3}\cos 2\theta_\lambda \vec{S}_\lambda + \frac{2}{3}\cos 2\theta_c \vec{S}_c\right)$$

This last operation relates the angle θ_c to the decay rate $\Gamma(\psi \to \eta_c + \gamma)$. Despite the fact that there is not even yet definite evidence for the existence of η_c particle, the measured branching ratio for the chain

$$\psi \to \eta_c(2800) + \gamma$$
$$\hookrightarrow \gamma + \gamma$$

requires, with reasonable assumptions on the η_c decays, that

$$\theta_c \simeq 45^\circ \ .$$

From this value one can compute all the weak charges with $\Delta c = 1$.

$$\langle K^- | Q^{13+i14} | D^0 \rangle = -1.12$$

$$\langle \eta | Q^{13+i14} | F^+ \rangle = .96$$

$$\langle \overline{K^{*0}} | Q_5^{13+i14} | D^+ \rangle = \langle \phi | Q_5^{13+i14} | F^+ \rangle = .29$$

$$\langle \eta^- | Q^{11+i12} | D^0 \rangle = \langle K^0 | Q^{11-i12} | F^+ \rangle = .91$$

$$-\sqrt{2} \langle \rho^0 | Q_5^{11+i12} | D^+ \rangle = \langle K^{*0} | Q_5^{11+i12} | F^+ \rangle = .42$$

B. A RELATIVISTIC MASS FORMULA FOR VECTOR MESONS

In order to obtain a relativistic description of interacting particles, we use the approach of Bakamjian and Thomas, itself reconsidered by Coester. We will deal with wave functions defined in momentum space. As we all know from the work of Currie, Jordan and Sudarshen, a relativistic description of interacting particles in terms of coordinates in space-time has been unsuccessful.

So let us consider two noninteracting particles of mass m_1 and m_2 and spin s_1 and s_2, and let us define the variables:

$$p = p_1 + p_2 \quad \text{with } p_1^2 = m_1^2,\ p_2^2 = m_2^2$$

$$k = 1/2 L(p)^{-1}(p_1 - p_2)$$

$L(p)$ being the relativistic boost mapping $p^0 = (\sqrt{p^2}, \vec{0})$ into p and leaving invariant the vectors orthogonal to p^0 and p.

One can rewrite the generators of the Poincare group as:

i) Translations: $\vec{P} = \vec{p}$, $M_0 = \sqrt{\vec{p}^{\,2} + M_0^2}$

with $M_0 = \sqrt{\vec{k}^{\,2} + m_1^2} + \sqrt{\vec{k}^{\,2} + m_2^2}$

ii) Rotations: $\vec{J} = \vec{x} \times \vec{p} + \vec{y} \times \vec{k} + \vec{S}$

with $\vec{x} = i\dfrac{\partial}{\partial \vec{p}}$, $\vec{y} = i\dfrac{\partial}{\partial \vec{k}}$ and $\vec{S} = \vec{S}_1 + \vec{S}_2$ where $\vec{S}_1$ and $\vec{S}_2$ are the spin representations for particles (1) and (2) respectively.

iii) Boosts : $\vec{K} = 1/2\left[\vec{x}H_0 + H_0\vec{x}\right] - (\vec{y}\times\vec{k}+\vec{S})\times\vec{p}(M_0+H_0)^{-1}$

As one can see, if we replace in the above expressions the mass M_0 by a "mass" M depending only on scalars made with $\vec{y}$, $\vec{k}$ and the spin variables, the commutation relations of the new generators are still the same. In doing so, we introduce some kind of an interaction between the particles compatible with Poincare symmetries.

In particular, we can exhibit a harmonic oscillator like spectrum in order to describe mesons as quark-antiquark bound systems. Taking as an only guide simplicity, we shall define the mass operator M in the very naive form:

$$M = (\vec{k}^2 + \omega^2\vec{y}^2 - E_0 + m_1^2)^{1/2} + (\vec{k}^2 + \omega^2\vec{y}^2 - E_0 + m_2^2)^{1/2} .$$

As we see ω plays the role of a coupling constant, and M becomes M_0 when the coupling constant of the "interaction" is put to be zero. The constant E_0 is set to be the energy of the ground state for the harmonic oscillator while m_1 and m_2 are to be connected to the quark masses. The eigenvalues of M are then:

$$M_u = (2n\omega + m_1^2) + (2n\omega + m_1^2)^{1/2} \quad n = 0,\ 1,\ 2,\ \ldots$$

As we see, we did not introduce terms connected with spin-spin and spin-orbit couplings. This could be done in order to obtain a more elaborate and more complete mass formula. However, we will see that such a crude mass formula gives results in good agreement with experimental values for the vector mesons.

We shall assume that a quark and the corresponding antiquark have the same mass, and we will suppose $m_u = m_d$ as usual.

For isospin one mesons, this leads to a linear low in the $(\mathrm{mass})^2$ which has been frequently observed. More precisely, we get:

$$M^2_{n,\ I=1} = 4(2n\omega + m_u^2).$$

From the mass values of the $J^{PC} = 1^{--}$ $\rho(770)$ and $\rho(1600)$ mesons, corresponding to n = 0 and n = 2 respectively, and of the $J^{PC} = 1^{+-}$ $\phi(1020)$ and $\psi(3100)$ corresponding to n = 0, we deduce:

$\omega = 123 \times 10^3\ \mathrm{MeV}^2$; $m_u = 385$ MeV; $m_\lambda = 510$ MeV; $m_c = 1550$ MeV.

So, we find for the 1^{+-}, n = 1, B(1220) meson the mass of 1255 MeV, for the 1^{--}, n = 0, K)892) meson the mass of 895 MeV and for the 1^{+-}, n = 1, K(1320) the mass of 1338 MeV.

We note that in this approach the excited state $\psi'(3700)$ corresponds to n = 4; for n = 2, we obtain the corresponding mass of 3400 MeV.

We have in this way the mass of the charmed vector mesons $D^*(c\bar{u})$ and $F^*(c\bar{\lambda})$:

$$m_{D^*(n=0)} \simeq 1935 \text{ MeV}. \quad m_{D^*(n=1)} \simeq 2230 \text{ MeV}.$$

$$m_{F^*(n=0)} \simeq 2060 \text{ MeV}. \quad m_{F^*(n=1)} \simeq 2345 \text{ MeV}.$$

Thus this simple mass formula for vector mesons predicts masses in a fairly good agreement with the experimental values. Let us emphasize on the elementary expression we have chosen for the mass operator. One can hope to obtain on one hand a more general formula for mesons by introducing some kind of spin-spin or spin-orbit couplings, and on the other hand, a generalization of such a mass formula for baryons. We can remark we find again here the advantages of the Gell-Mann Okubo formula and its generalization to SU(4): SU(4) and SU(3) breakings appear here by the quark mass difference: $m_u = m_d < m_\lambda << m_c$.